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# A bathtub model for the star-forming interstellar medium

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**Abstract.** The bathtub model of the star forming interstellar medium (ISM) is based on the powerful constraint that mass has to be conserved when gas flows through its various thermal and density phases, ending up eventually in a young star or being blown away by stellar feedback. It predicts that the star formation rate of a molecular cloud is not determined by the cloud's mass or its internal collapse timescale, but rather by the accretion rate of new gas. For the most simple case of a constant accretion flow an equilibrium state is reached quickly where the star formation rate equals the accretion rate and where the dense gas mass is constant and independent of time. The mass of the young star cluster, on the other hand, increases linearly with time. The stellar mass fraction therefore represents a sensitive clock to measure the age of the star-forming region. The bathtub models predicts that the efficiency of star formation is small, of order 1%, even in the dense filamentary phases of molecular clouds and for the large gas depletion timescales of star-forming galaxies of order  $5 \times 10^8 - 10^9$  yrs.

Key words. Stars: formation - ISM: clouds - ISM: kinematics and dynamics

#### 1. Introduction

Stars form from cold and dense molecular gas. The star formation rate (SFR), averaged over a whole galaxy, correlates well with the total molecular gas mass  $M_{H_2}$  with a surprisingly universal depletion timescale  $\tau_{depl} = M_{H_2}/SFR \approx 10^9$  yrs (Tacconi et al. 2017).

On small scales, however, the physical properties of molecular clouds and as a result their star formation histories vary strongly from place to place. Francesco Palla's work has been pioneering in outlining the complexity of local star formation and in providing insight into properties of star forming regions, like their typical star formation timescales which are of order  $2 - 4 \times 10^6$  yrs and their star formation histories (Palla & Stahler 1999). In

order to explain the inefficiency of star formation Lada et al. (2010), (see also Burkert & Hartmann 2013) suggested that there exists a density threshold of order  $n \approx 10^4$ cm<sup>-3</sup> for star formation. Lee & Hennebelle (2016) termed this dense star-forming region within a molecular cloud the gaseous protocluster. Interestingly, observations indicate that the mass fraction of dense gas is rather universal with a value of 10% the total molecular cloud mass (Lada et al. 2010; Vutisalchavakul et al. 2016). Even more interesting is the fact that even in this dense gas phase the stellar mass fraction is small, of the order of 10%.

One serious observational caveat in unraveling the star formation history of a molecular cloud is the fact that the presently observed young stellar population formed under physical conditions that are not directly observable anymore. The current structure of the molecular cloud, on the other hand, determines future star formation which is not yet observable. Only in an equilibrium state can the past be directly related to the future. One would probably at first focus on the available dense molecular gas as the primary site of star formation. This concept is however misleading as molecular clouds and especially their dense gas regions are not isolated. As shown below it is actually not the amount of dense gas or its gravitational collapse timescale that determines the star formation rate, but rather the rate of dense gas formation.

Here I can only introduce the most simple version of the ISM bathtub model (see also Bouche, N. et al. 2010, for the galactic bathtub and Burkert & Hartmann 2013). A detailed investigation of more complex situations, including gravity driven accretion and exponentially increasing star formation rates will be discussed in a subsequent paper (Burkert, in preparation).

# 2. The simple bathtub model (SBT) of the star-forming ISM

Let us consider a star-forming region that is powered by a constant inflow of gas  $\dot{M}_{acc}$ . In addition, we apply assumptions, often also used in numerical simulations, of a critical gas density threshold  $n_{dense} = 10^4$  cm<sup>-3</sup> (Lada et al. 2010), above which stars form on a local free fall timescale (Krumholz et al. 2012)  $\tau_{ff,dense} \approx 4 \times 10^5$  yrs and a star formation rate that correlates linearly with the dense gas mass, multiplied by a constant efficiency per free-fall time  $\epsilon_{ff}$ :

$$\frac{dM_{dense}}{dt} = \dot{M}_{acc} - \frac{M_{dense}}{\tau_{ff,dense}} \tag{1}$$

$$SFR = \frac{dM_*}{dt} = \epsilon_{ff} \frac{M_{dense}}{\tau_{ff,dense}}$$
(2)

Without gas accretion, i.e.  $\dot{M}_{acc} = 0$ , the solution is  $M_{dense} = M_0 \times \exp(-t/\tau_{ff,dense})$  and the

dense gas mass and SFR decreases exponentially with time. This solution suffers from the fact that there is no explanation how the initial dense gas reservoir  $M_0$  was generated and why star formation did not already start during that phase. In addition, as shown e.g. by Palla & Stahler (1999), the star formation rates in some star-forming region are actually *increasing* exponentially with time which is just the opposite of the solution that we get without accretion.

Including a non-zero accretion term the solution is fundamentally different:

$$M_{dense} = \dot{M}_{acc} \tau_{ff,dense} \left[ 1 - \exp\left(-\frac{t}{\tau_{ff,dense}}\right) \right]$$
$$SFR = \epsilon_{ff} \dot{M}_{acc} \left[ 1 - \exp\left(-\frac{t}{\tau_{ff,dense}}\right) \right] \quad (3)$$

We can now identify two phases. For t<  $\tau_{ff,dense}$  the dense gaseous protocluster mass increases linearly with time  $M_{dense} = \dot{M}_{acc} \times t$ , independent of  $\tau_{ff,dense}$ , as does the SFR. This is the non-equilibrium filling phase of the bathtub. The situation changes for t>  $\tau_{ff,dense}$ . The system now approaches a constant dense gas mass  $M_{dense} = \dot{M}_{acc} \times \tau_{ff,dense}$  and a constant star formation rate  $SFR = \epsilon_{ff} \times \dot{M}_{acc}$ , despite the fact that gas is still continuously being accreted. In this equilibrium phase, the SFR does not depend at all on the amount of dense gas M<sub>dense</sub>, nor on the physics of star formation, hidden in  $\tau_{ff,dense}$ . It however depends critically on the physics of accretion. This demonstrates that it is the flow of gas through the various gas phases of the ISM that determines its star formation rate as function of time and not the instantaneous amount of dense gas that is observed at any given time. Indeed, being constant and time independent, the dense gas mass does not contain any information about the past or future star formation history. Most observed star forming regions have ages of order a few Myrs, which is much longer than  $\tau_{ff,dense}$ . They should therefore be in this equilibrium phase.

## 3. The diffuse molecular gas reservoir and the dense gas fraction

Despite being the primary site of star formation, the dense molecular gas phase represents only a small fraction of the galactic molecular gas and is surrounded by a diffuse molecular gas reservoir with densities of order  $n_{diff} \approx 10^2 \text{ cm}^{-3}$  and corresponding free fall timescales of  $\tau_{diff} \approx 4 \times 10^6$  yrs. It has long been assumed that this diffuse component is supported against gravitational collapse by internal turbulent flows. New observations and theoretical models however indicate that the observed velocities are actually signatures of gravitational collapse onto the embedded dense, star-forming gas regions (e.g. Ballesteros-Paredes et al. 2011). In this case, the diffuse component is an ideal candidate for the gas reservoir that feeds the dense gas (Mac Low et al. 2017). Let us now investigate the implication of this scenario within the SBT model. We approximate the infall rate of diffuse gas by  $M_{acc,dense} = M_{diff} / \tau_{ff,diff}$ . In the equilibrium phase, discussed above, we then find

$$M_{dense} = \dot{M}_{acc,dense} \tau_{ff,dense} = (4)$$
$$= M_{diff} \times \frac{\tau_{ff,dense}}{\tau_{ff,diff}} = 0.1 M_{diff}$$

which is in excellent agreement with observations. Note that the dense gas fraction depends critically on the ratio of the collapse timescales of the diffuse and dense gas component which could vary strongly with galactic environment.

### Origin of the large gas depletion timescale in galaxies

In the SBT equilibrium phase the dense gas mass and the SFR is constant. The stellar mass is however linearly increasing with time

$$M_* \sim \epsilon_{ff} \dot{M}_{acc} \times t.$$
 (5)

 $M_*/M_{dense}$  therefore is very sensitive to the evolutionary time t of the star forming region and in late phases could in principle approach values of unity or larger. For typical

local star-forming regions in the solar neighborhood  $M_* \approx 0.1 M_{dense}$ . It can be shown (Burkert, in preparation) that for typical ages of star forming regions  $t \approx 10 \times \tau_{ff,dense}$  this implies that even in the dense gaseous protocluster environment star formation must be rather inefficient with values  $\epsilon_{ff} \approx 0.01$ . This now allows us to determine a molecular gas depletion timescale. With  $M_{H_2} = M_{diff} + M_{dense} \approx M_{diff}$  and using the previous equations we find

$$\tau_{depl} = \frac{M_{H_2}}{SFR} = \frac{\tau_{ff,diff}}{\epsilon_{ff}}$$

$$\approx 100 \times \tau_{ff,diff} \approx 4 \times 10^8 yrs$$
(6)

In the framework of the SBT model it is therefore the combination of the relatively large collapse timescale of the diffuse molecular gas, combined with the low efficiency of star formation in the dense molecular gas component that determines the global gas depletion timescales of galaxies.

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